



Short communication

Variational iteration method for Bratu-like equation arising in electrospinning

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ABSTRACT

This paper points out that the so called enhanced variational iteration method (Colantoni & Boubaker, 2014) for a nonlinear equation arising in electrospinning and vibration-electrospinning process is the standard variational iteration method. An effective algorithm using the variational iteration algorithm-II is suggested for Bratu-like equation arising in electrospinning. A suitable choice of initial guess results in a relatively accurate solution by one or few iteration.

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1. Introduction

As Galileo Galilei said that “mathematics is the language with which God wrote the universe,” a mathematical model is also much needed for electrospinning and its modifications, such as the vibration-electrospinning (He, Liu, Mo, Wan, & Xu, 2008), Bubbil spinning which is to use polymer bubbles or polymer membranes for fabrication of nano or micro materials including nanofibers (He et al., 2012; Kong, He, & Chen, 2013), superfine film (Dou, Liu, Wang, & He, 2013), crimped nanofibers (Chen, Zhang, Kong, He, & Chen, 2013), and nanoparticles (Dou & He, 2012). Some useful models were appeared in open literature (e.g., Xu, Wang, & Faraz, 2011), among which Wan–Guo–Pan model is the most famous (Colantoni & Boubaker, 2014; Wan, Guo, & Pan, 2004), which reads

$$\frac{\partial^2 v}{\partial z^2} + \lambda e^v = 0 \quad (1)$$

where $v = -6 \ln u$, u is the velocity of the jet in the spinning process. Colantoni and Boubaker (2014) suggested the following variational

iteration algorithm by the so-called enhanced variational iteration method

$$v_{n+1}(z) = v_n(z) + \int_0^z \left\{ (s-z) \left[\frac{\partial^2 v_n(s)}{\partial s^2} + \lambda e^{v_n(s)} \right] \right\} ds \quad (2)$$

however this is nothing new but the variational iteration method itself (He & Wu, 2007).

2. Variational iteration method

The variational iteration method was first proposed in later 1990s (He, 1998, 1999), and was further developed into an effective method for various nonlinear equations (He, 2006, 2007, 2008, 2012; He & Wu, 2007). To illustrate the method, we consider a general nonlinear equation in the form

$$\frac{\partial^2 v}{\partial z^2} + f(v) = 0 \quad (3)$$

where $f(v)$ is a nonlinear function of v .

The variational iteration algorithm reads (He & Wu, 2007)

$$v_{n+1}(z) = v_n(z) + \int_0^z \left\{ (s-z) \left[\frac{\partial^2 v_n(s)}{\partial s^2} + f(v_n(s)) \right] \right\} ds \quad (4)$$

Letting

$$f(v) = -2e^v \quad (5)$$

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we obtain the variational iteration algorithm

$$v_{n+1}(z) = v_n(z) + \int_0^z \left\{ (s-z) \left[\frac{\partial^2 v_n(s)}{\partial s^2} - 2e^{v_n(s)} \right] \right\} ds \quad (6)$$

This is Eq. (25) obtained by Colantoni and Boubaker (2014), which was given a new name, the enhanced variational iteration method, however, it is the standard variational iteration method. Application of the variational iteration method to the Bratu equation was done by Batiha (2010) and Saravi, Hermann, and Kaiser (2013), respectively.

3. Variational Iteration method for Liu–Wang equation

Liu and Wang (2014) pointed out that λ in Eq. (1) is not a constant for electrospinning, vibration electrospinning or bubble electrospinning, it should be written in the form

$$\frac{\partial^2 v}{\partial z^2} + \lambda(v)e^v = 0 \quad (7)$$

or

$$\frac{\partial^2 v}{\partial z^2} + \frac{18E^2(I - (QkE/(\rho \exp(-v/6))))^2}{Q^2 \exp(v/3)} e^v = 0 \quad (8)$$

where z is axial coordinate, $v = -6 \ln u$, u is the axial velocity, E is the applied voltage, I is the current, Q is the flow rate, k is the conductivity, ρ is the density.

Liu and Wang (2014) called Eq. (8) as Bratu-like equation, however, Eq. (8) is beyond the Bratu equation, and it is referred as Liu–Wang equation, which is a development of Wan–Guo–Pan equation (Wan et al., 2004).

The variational iteration algorithm for Liu–Wang equation can be expressed in the form

$$v_{n+1}(z) = v_n(z) + \int_0^z \left\{ (s-z) \left[\frac{\partial^2 v_n(s)}{\partial s^2} + \frac{18E^2(I - (QkE/(\rho \exp(-v_n(s)/6))))^2}{Q^2 \exp(v_n(s)/3)} e^{v_n(s)} \right] \right\} ds \quad (9)$$

Eq. (9) is called as the variational iteration algorithm-I. The variational iteration algorithm-II is much simpler for Liu–Wang equation, which reads (He, 2012)

$$v_{n+1}(z) = v_0(z) + \int_0^z \left\{ (s-z) \frac{18E^2(I - (QkE/(\rho \exp(-v_n(s)/6))))^2}{Q^2 \exp(v_n(s)/3)} e^{v_n(s)} \right\} ds \quad (10)$$

where $v_0(z)$ is the initial solution which should satisfy the initial conditions of Eq. (8).

A suitable choice of $v_0(z)$, one iteration or few iterations might result in a relative accuracy of the solution, see examples given by Khan, Vázquez-Leal, Hernandez-Martínez, and Faraz (2012) and Faraz (2011).

4. Conclusion

We conclude that Eq. (1) with a constant λ is invalid for description of the electrospinning process; the enhanced

variational iteration method used by Colantoni and Boubaker (2014) is the standard variational iteration method itself, not any enhancement was made at all.

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